Influence of Large Amplitude on Response to Sonic Booms

N.K. Adi Murthy* and R.S. Alwar†
Indian Institute of Technology, Madras, India

Theme

CONSIDERABLE amount of literature exists on the linear dynamic response of discrete and continuous systems subjected to sonic booms.[1-3] In the present investigation the influence of large amplitude on the dynamic response of rectangular isotropic plate with special reference to glass panel subjected to sonic boom is studied for several boom parameters like overpressure, waveform duration, and rise time. The solution procedure adopted in the present investigation consists of expressing the governing nonlinear differential equations in the rate form, a method adopted by the authors earlier,[4] and in the process these equations are reduced to a set of linear differential equations. These linear equations are solved approximately spacewise by Galerkin method and time-wise using the Houbolt[5] scheme. The influence of nonlinearity is presented in the form of a ratio of maximum linear to maximum nonlinear dynamic response. It should be mentioned that the rate form linearization has a specific advantage that the resulting linear equations are themselves exact without any approximations, and it can be used equally well for both static and dynamic nonlinear problems. An investigation on nonlinear dynamic behavior of shells of revolution using a similar technique has recently appeared.[6]

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Differentiating the governing nonlinear differential equations[7] with respect to time, the linear differential equations in the rate form can be written as

$$(W_{,x})\bar{W}_{,xx}+\left(\frac{1-\nu}{2}W_{,x}\right)\bar{W}_{,yy}+\left(\frac{1+\nu}{2}W_{,y}\right)\bar{W}_{,xy}$$
$$+\left(W_{,xx}+\frac{1-\nu}{2}W_{,yy}\right)\bar{W}_{,x}+\left(\frac{1+\nu}{2}W_{,xy}\right)\bar{W}_{,y}+\bar{U}_{,xx}$$
$$+\left(\frac{1-\nu}{2}\right)\bar{U}_{,yy}+\left(\frac{1+\nu}{2}\right)\bar{V}_{,xy}=0 \tag{1}$$

$$\left(\frac{1-\nu}{2}W_{,y}\right)\bar{W}_{,xx}+(W_{,y})\bar{W}_{,yy}+\left(\frac{1+\nu}{2}W_{,x}\right)\bar{W}_{,xy}$$
$$+\left(\frac{1+\nu}{2}W_{,xy}\right)\bar{W}_{,x}+\left(W_{,yy}+\frac{1-\nu}{2}W_{,xx}\right)\bar{W}_{,y}$$
$$+\left(\frac{1-\nu}{2}\bar{V}_{,xx}+\bar{V}_{,yy}+\left(\frac{1+\nu}{2}\right)\bar{U}_{,xy}=0\right. \tag{2}$$

$$\bar{W}_{,xxxx}+2\bar{W}_{,xxyy}+\bar{W}_{,yyyy}-(12/h^2)$$
$$[\{U_{,x}+\tfrac{1}{2}W_{,x}^2+\nu(V_{,y}+\tfrac{1}{2}W_{,y}^2)\}\bar{W}_{,xx}$$
$$+\{V_{,y}+\tfrac{1}{2}W_{,y}^2+\nu(U_{,x}+\tfrac{1}{2}W_{,x}^2)\}$$
$$\bar{W}_{,yy}+(1-\nu)\{U_{,y}+V_{,x}+W_{,x}W_{,y}\}\bar{W}_{,xy}$$
$$+\{W_{,x}W_{,xx}+\nu W_{,x}W_{,yy}+(1-\nu)W_{,y}W_{,xy}\}$$
$$\bar{W}_{,x}+\{W_{,y}W_{,yy}+\nu W_{,y}W_{,xx}+(1-\nu)W_{,x}W_{,xy}\}\bar{W}_{,y}$$
$$+\{W_{,xx}+\nu W_{,yy}\}\bar{U}_{,x}+\{(1-\nu)W_{,xy}\}\bar{U}_{,y}$$
$$+\{(1-\nu)W_{,xy}\}\bar{V}_{,x}+\{W_{,yy}+\nu W_{,xx}\}\bar{V}_{,y}]$$
$$=1/D\{\bar{q}-\rho h\bar{W}_{,tt}\} \tag{3}$$

where $2a$, $2b$=sides of plate, h=thickness of plate, E= modulus of elasticity, ν=Poisson's ratio, ρ=mass density, t=time, $D=Eh^3/12(1-\nu^2)$, W, U, V=transverse and in-plane displacements, q=forcing function, $\bar{W}=dW/dt$, $\bar{U}=dU/dt$, $\bar{V}=dV/dt$ and $\bar{q}=dq/dt$.

The boundary conditions for a simply-supported plate with immovable edges are:

$$\bar{U}=\bar{W}=\bar{W}_{,xx}=0 \text{ along } x=0,\ 2a \text{ and } \bar{V}$$
$$=\bar{W}=\bar{W}_{,yy}=0 \text{ along } y=0,\ 2b \tag{4}$$

An approximate solution to Eqs. (1) to (3) is obtained by assuming

$$\bar{W}=W_0\sin\frac{\pi x}{2a}\sin\frac{\pi y}{2b} \tag{5}$$

$$\bar{U}=\frac{C_W W_0\pi}{16a}\left(\cos\frac{\pi y}{b}-1+\nu\lambda^2\right)\sin\frac{\pi x}{a} \tag{6}$$

$$\bar{V}=\frac{C_W W_0\pi}{16b}\left(\cos\frac{\pi x}{a}-1+\frac{\nu}{\lambda^2}\right)\sin\frac{\pi y}{b} \tag{7}$$

where λ=aspect ratio, and C_W and W_0 represent the transverse displacement and velocity at the center of the plate, respectively. The previous choice of functions for the new dependent variables viz. the velocities satisfy all the boundary conditions given by Eq. (4) and also it can be verified that the chosen functions satisfy Eqs. (1) and (2) exactly.

Expressing $(\bar{W}_{,tt})$ in Houbolt backward difference form in Eq. (3), and applying Galerkin method at any time step J, the following linear algebraic equation is obtained.

$$\left[\frac{1}{12}(1+\lambda^2)^2+\frac{1}{32}\{2(1+12\nu)\lambda^2+(21-8\nu^2)(1+\lambda^4)\}\right.$$
$$\left.\left(\frac{C_W}{h}\right)^2\right]\left(\frac{W_0^J}{h}\right)-\frac{4a^4}{3D\pi^4h}\left[\frac{16\bar{q}}{\pi^2}-\frac{\rho h}{(\Delta t)^2}\right.$$
$$\left.(2W_0^J-5W_0^{J-1}+4W_0^{J-2}-W_0^{J-3})\right]=0 \tag{8}$$

where Δt=time increment.

The value of C_W is evaluated to start with using the initial conditions and subsequently from the following relation

$$(C_W)_J=(C_W)_{J-1}+(W_0^J)(\Delta t) \tag{9}$$

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*Research Scholar, Department of Applied Mechanics.
†Professor, Department of Applied Mechanics.

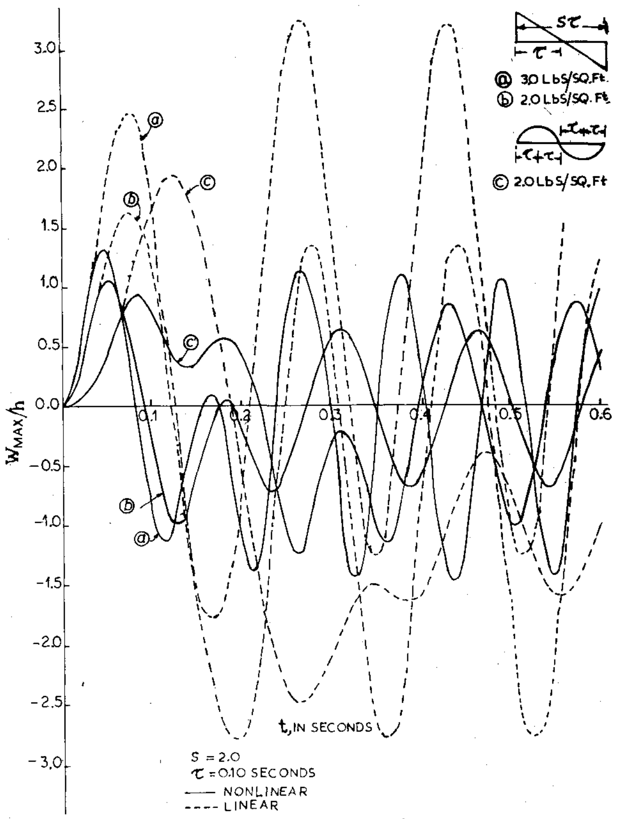

Fig. 1 Effect of overpressure and waveform on the response.

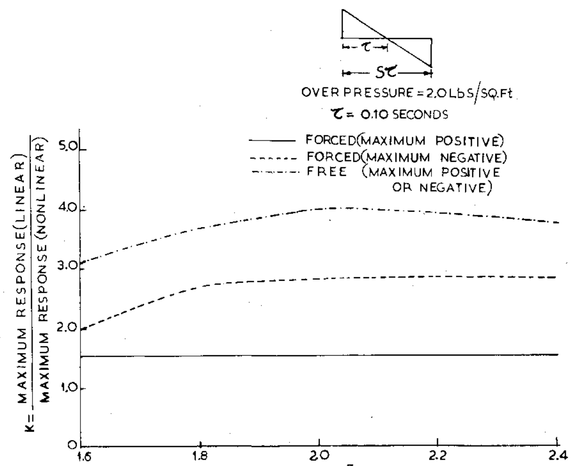

Fig. 2 Variation of K with respect to S.

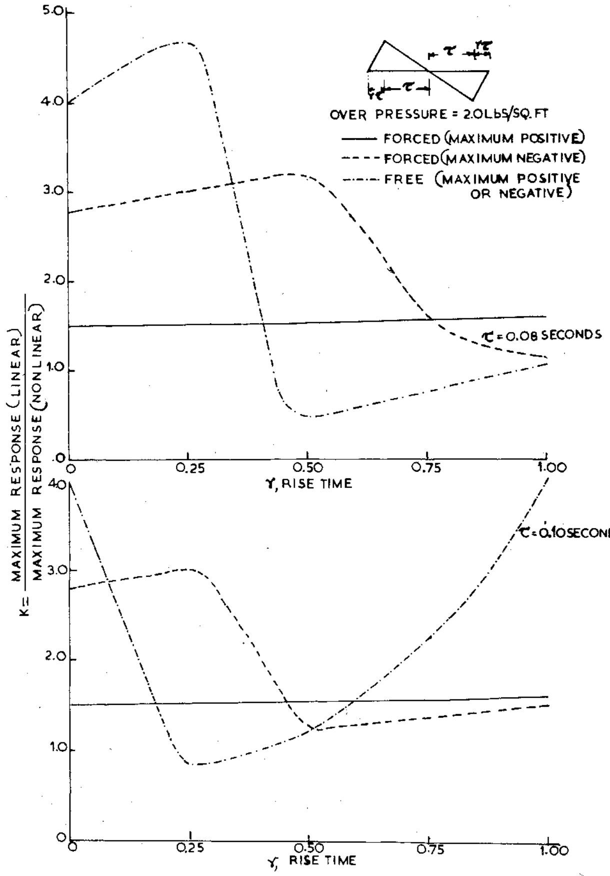

Fig. 3 Variation of K with respect to rise time r for a) $t = 0.08$ sec and b) $t = 0.10$ sec.

The influence of nonlinearity on the response of glass window panel to sonic boom is studied for the following data: $2a = 96.00''$, $h = 0.25''$, $E = 10^7$ psi, $\nu = 0.23$, $\lambda = 1.0$, $\rho = 236 \times 10^{-6}$ lbs sec^2 /in^4.

The effect of overpressure on the linear and nonlinear response of a simply-supported glass window panel subjected to symmetric '*N*' wave and complete cycle sine wave pulse is shown in Fig. 1. It can be seen that for an overpressure of 2.00 lbs/sq. ft. and positive pulse duration of 0.10 sec, which are the values of a medium aircraft flying at normal cruising altitudes, the maximum dynamic deflection is of the order ot thickness of the panel and hence in the nonlinear range. It can also be observed that the time necessary for the positive and negative response to reach their maximum decreases with increasing overpressure. Figure 2 shows the variation of K, with respect to S, K being the ratio of maximum linear dynamic response to maximum nonlinear dynamic response. It is clear from this Fig. 2 that the value of K for the three phases of motion is nearly constant for values of S ranging from 1.6 to 2.4.. With further increase in S, K which represents the nonlinearity effect on the response, changes relatively faster for free motion compared to forced motion, where K remains essentially constant.

The variation of K with respect to rise time has been shown in Fig. 3 for two typical pulse durations. It can be observed that there is no appreciable change in K with respect to rise time during the positive forced motion. During negative forced motion K increases with rise time initially and reaches maximum for a total pulse duration of about 0.25 sec in both the cases. Also it can be seen that during negative forced motion, K takes a maximum value of about 3.0. During free motion K takes a maximum value of 4.60 for $\tau = 0.08$ sec and has a value of around 4.00 for $\tau = 0.10$ sec. The minimum value of K is less than unity and occurs at the same rise time when K is maximum during negative forced motion.

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